The first derivative of σ_{zz} with respect to z determines σ_{zx} and σ_{zy}, and the combination of σ_{zz} and its second derivative with respect to z determines the remaining components of the stress. These two functions must satisfy the condition that the integral (3.2) must vanish on the contour Γ. This condition is a consequence of the assumption that the stressed state does not contain a normal rotation (G = 0) and can be used in the algorithm in order to reduce the errors of the starting measurements.

Problems formulated in this manner have thus far been solved only for the axisymmetric stressed state [4, 7]. The stressed state not containing normal rotation is significantly more extensive and contains an axisymmetric state as a particular case.

In conclusion it is my pleasant duty to thank Kh. K. Aben for proposing the subject and also for his constant well-meaning interest in this work.

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DYNAMIC DUCTILITY PEAK WITH HIGH-VELOCITY FAILURE OF METAL SHELLS

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An explanation of the dynamic ductility peak [1] is given. It is shown that this effect is connected with a sharp deterioration in metal ductility properties with a strain rate of $\dot{\varepsilon} \sim 10^5$ sec^{-1}.

The failure of cylindrical metal shells expanding under the action of detonation products with high strain rates of $\dot{\varepsilon} > 10^4$ sec^{-1} was studied in [1-4]. Here it was detected that high-velocity failure exhibits a number of features which relate to existence of a scale effect and a dynamic ductility peak.

An explanation of these features will be sought within the scope of describing failure as a two-stage process [2]. The first stage consists of damage accumulation with plastic flow. In the second stage by crack propagation there is separation of the shell parts due to stored elastic energy.

We divide the process of damage accumulation into two stages. We assume that in the first stage there is accumulation of point defects, and in the second there is growth of pores which are the result of merging of point defects. Similar to [5] we shall assume that defects arise with unconservative movement of steps which form from the intersection of edge and screw dislocations. Then from [5] it follows that the concentration of defects $c_d = f(\varepsilon)$. Since occurrence of pores occurs with some critical concentration of them c_d^*, the material should experience some strain ε_0 prior to pore growth commencing.

In the second stage pore growth is determined by the viscosity and inertial properties of the material. This assumption is correct with high strain rates. According to [6] the equation for pore radius has the form

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$$\dot{R}/R = (\sigma_m - \sigma_0)/\eta_1, \tag{1}$$

where η_1 is a constant value with a dimension of viscosity; σ_0 is threshold stress; $\sigma_m = \sigma_{ii}/3$ is tensile spherical part of the stress tensor. From Eq. (1) we find the characteristic time for pore growth

$$t_p = \eta_1/(\sigma_m - \sigma_0). \tag{2}$$

Taking this into account we write an equation for failure strain as

$$\varepsilon_p = \int_0^{t_1} \dot{\varepsilon}\, dt + \int_{t_1}^{t^*} \dot{\varepsilon}\, dt. \tag{3}$$

Assuming that $\varepsilon_0 = \int_0^{t_1} \dot{\varepsilon}\, dt$ and considering that $\dot{\varepsilon}$ = const for $\varepsilon > \varepsilon_0$, we find that

$$\varepsilon_p = \varepsilon_0 + \dot{\varepsilon} t_p \tag{4}$$

($t_p = t^* - t_1$). The assumption of $\dot{\varepsilon}$ = const corresponds to the fact that the shell expands with a constant velocity with $\varepsilon > \varepsilon_0$. As will be shown below, with $\varepsilon > \varepsilon_0$ pressure in the detonation products is small, and therefore acceleration of the shell may be ignored. In deriving Eq. (4) the time for crack propagation through the shell was ignored since as will be shown below it is an order of magnitude less than the time for failure t^*.

In order to determine σ_m we consider the stressed state of a shell within the scope of the Prandtl–Reis elastoplastic model [7, 8]:

$$\sigma_i = -p + S_i,\ S_i = S_i' \sqrt{\tfrac{2}{3}}\, Y \Big/ \sqrt{\sum_{i=1}^{3} (S_i')^2},\quad \dot{S}_i' = 2\mu(\dot{\varepsilon}_i - 1/3\dot{V}/V),\quad \dot{V}/V = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3,\ i = 1, 2, 3 \tag{5}$$

(σ_i, S_i, ε_i, p, V, μ, Y are principal stresses, stress deviators, strains, pressure, specific volume, shear modulus, and yield strength). Equations (5) should be supplemented by equations of movement and energy. The complete set of equations is given for example in [7, 8]. In the case of plane strain for a shell the following relationships are valid

$$\varepsilon_3 = 0,\ \dot{V}/V \ll \dot{\varepsilon}_1,\ \dot{V}/V \ll \dot{\varepsilon}_2. \tag{6}$$

By substituting Eqs. (6) in set (5) we obtain

$$\varepsilon_2 \approx -\varepsilon_1,\ \varepsilon_2 > 0,\ S_3' \approx 0,\qquad S_2' \approx -S_1' \approx 2\mu\varepsilon_2,\ S_2 = Y/\sqrt{3},\qquad S_1 = -Y/\sqrt{3},\ \sigma_2 - \sigma_1 = \frac{2}{\sqrt{3}} Y, \tag{7}$$

whence

$$\sigma_m = (\sigma_1 + \sigma_2)/2. \tag{8}$$

The value of σ_1 varies from $-p_d$ (at the inner shell surface) to zero (at the outer shell surface), and therefore $|\sigma_1| < p_d$ [p_d is pressure in the detonation products (DP) in the vicinity of the shell]. We evaluate p_d in the range $t_1 < t < t^*$. Assuming that DP expansion is adiabatic, we find that $p_d = p_0(R_0/a)^{2\gamma}$, $a = (1 + \varepsilon_0)a_0$, $p_0 = p_i/2$ (R_0, a_0 are initial DP radius and internal shell radius). By substituting data from [3]: $R_0/a_0 \simeq 1/2$, $\gamma = 3$, p_i = 25.2 GPa (TG50/50), $\varepsilon_0 \simeq 0.3$, we obtain $p_d \simeq 0.04$ GPa, $|\sigma_1| < p_d$. Since for material St. 20 [3] Y ≈ 0.4 GPa, from the last equation of (7) we find that $\sigma_1 \gg \sigma_2$, $\sigma_2 \sim Y$. By substituting these relationships in Eq. (8) we have

$$\sigma_m \approx \sigma_2/2 \approx Y/\sqrt{3}. \tag{9}$$

Equation (9) and the assumption v = const when $\varepsilon > \varepsilon_0$ ($t_1 < t < t^*$) are confirmed by numerical calculations in a computer. The problem was solved in the unidimensional case for the geometry shown in Fig. 1. The shell was described by a Prandtl–Reis elastoplastic model, and the DP were described by a model of an ideal gas with $\gamma = 3$. Calculations were carried out by a "cross" difference scheme using a mobile Euler grid. The calculation procedure is described in more detail in [8, 9]. The shell material was St. 20 with the parameters ρ = 7.8 g/cm³, E = 200 GPa, μ = 84 GPa, Y = 0.43 GPa (E is Young's modulus). Shell internal radius a_0 = 2.1 cm, and external radius b_0 = 2.19 cm. The detonation products at instant t = 0 were found in a cylinder with radius R_0 = 0.5 cm with the parameter $p_0 = p_i/2$, p_i = 25.2 GPa, ρ_0 = 1.65 g/cm³ (TG50/50). The radius of the cylinder with DP was selected somewhat less than in the experiment (R_0 = 0.75 cm) with the same shell velocity. This is connected with the fact that in calculations no consideration was given to outflow of DP through the shell ends. Given in Fig. 2 is the dependence $\sigma_m(\xi)$ (curve 1) calculated in a computer for $\dot{\varepsilon} = 3.72 \cdot 10^4$ sec⁻¹, ε = 0.56 [$\xi = (r - a)/(b - a)$, r is shell coordinate, a and b are internal

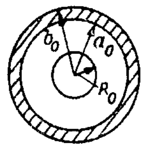

Fig. 1

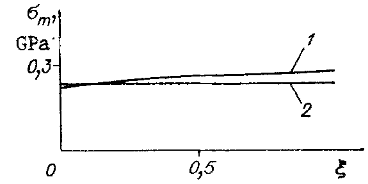

Fig. 2

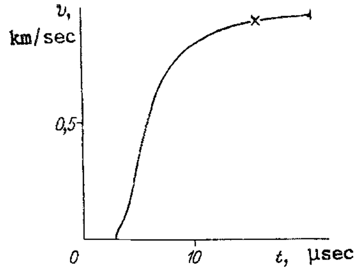

Fig. 3

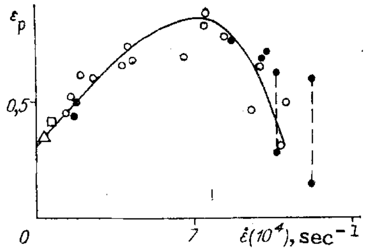

Fig. 4

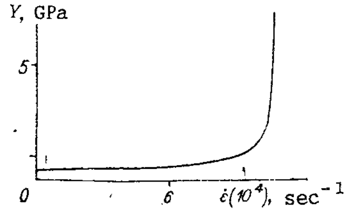

Fig. 5

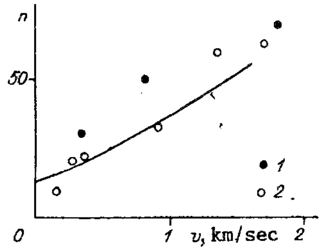

Fig. 6

and external shell surface coordinates]. It can be seen from Fig. 2 that $\sigma_m(r)$ changes weakly through the shell thickness and it agrees well with the $\sigma_m = 0.28$ GPa (line 2) found from Eq. (9). Shown in Fig. 3 is the calculated dependence of mean mass shell velocity on time. The cross relates to strain $\varepsilon_0 = 0.3$, and the vertical mark corresponds to the instant of shell separation. It can be seen that in the range $\varepsilon > \varepsilon_0$ shell velocity is almost constant. Assuming in (2) that $\sigma_0 = 0$ and taking account of (9), we find that

$$t_p = \sqrt{3}\eta_1/Y. \tag{10}$$

By substituting t_p in Eq. (4) we obtain the equation sought for ε_p:

$$\varepsilon_p = \varepsilon_0 + \sqrt{3}\eta_1\dot\varepsilon/Y. \tag{11}$$

We assume that the yield strength is a function of strain rate $Y = Y(\dot\varepsilon)$ and we use experimental results from [3] in order to determine the function $Y(\dot\varepsilon)$ and constants ε_0, η_1. Given in Fig. 4 are experimental results [3] (notation from [3]) and the curve $\varepsilon_p(\dot\varepsilon)$ approximating them, where $\varepsilon_p = (b_p - b_0)/b_0$, $\dot\varepsilon = v/b_0$, b_0 and b_p are shell radii with $t = 0$ and at the instant of failure (the dependence $\varepsilon_p(\log \dot e)$ ($\dot e = v/b_p$) was studied in [1-4]). Assuming that $Y = Y_0 = 0.43$ GPa with $\dot\varepsilon < 10^3$ sec^{-1}, from Fig. 4 and Eq. (11) we find that $\varepsilon_0 = 0.3$, $\eta_1 = 3.1 \cdot 10^3$ Pa·sec and the dependence $Y(\dot\varepsilon)$ given in Fig. 5.

A typical feature of the $Y = Y(\dot\varepsilon)$ curve obtained is the weak dependence of Y on $\dot\varepsilon$ with $\dot\varepsilon < 7 \cdot 10^4$ sec^{-1} and a sharp increase of Y for $\dot\varepsilon \sim 10^5$ sec^{-1}. A dependence is given in [10] for yield strength on shear rate for aluminum obtained in compression–shear tests. It can be seen that with $\dot\varepsilon \gtrsim 10^5$ sec^{-1} a strong increase is observed in the yield strength of aluminium, and the $Y(\dot\varepsilon)$ curve conforms qualitatively with that found in this work (Fig. 5). In the range $\dot\varepsilon < 7 \cdot 10^4$ sec^{-1} the dependence $Y(\dot\varepsilon)$ may be described within the scope of the viscoplastic model

$$Y = Y_0 + \eta\dot\varepsilon \tag{12}$$

($\eta = 3.42 \cdot 10^3$ Pa·sec is metal viscosity). Whence it follows that η_1 conforms with metal viscosity η.

In [11, 12] from experiments on explosive welding steel viscosity ws found in the range of interest to us 10^4 sec^{-1} $< \dot\varepsilon <$ 10^5 sec^{-1} which varies within the limits $\eta = (6\text{-}10) \cdot 10^3$ Pa·sec. This value exceeds η by a factor of two to three found in this work. The difference may be explained if it is assumed that viscosity depends not only on strain but also on the amount of strain $\eta = \eta(\dot\varepsilon, \varepsilon)$ [13]. The greater the strain, the greater the defect density, and therefore relaxation proceeds more rapidly and the importance of viscosity will be less. For example, in [13] in Fig. 1 it is shown that an increase in strain from 0.06 to 0.6 leads to a reduction in viscosity by a factor of three for Al 1100. In determining the viscosity of metals in [11, 12] total material displacement $S = \int_0^{l_2} z(y)\,dy$ is used which arises with welding by explosion (z is horizontal displacement, ℓ_2 is plate thickness). We estimate the value of average strain by the equation $\varepsilon \simeq S/\ell_2^2$. By substituting in this equation numerical values from [12]: ℓ_2 = 8 mm, S < 0.54 mm^2, we obtain the estimate sought $\varepsilon < 10^{-3}$. Since up to the instant of failure the steel undergoes essentially higher strain welded plates, then the viscosity obtained in this work is less than in [11, 12]. The difference noted above in viscosities is not connected with the effect of temperature since shell heating does not exceed 100 K ($\Delta T \simeq \sigma\varepsilon/(c_P\rho)$, $\varepsilon \simeq 0.6$, $\sigma \simeq Y \simeq 0.43$ GPa, $\rho = 7$ g/cm^3, $c_P = 0.46$ J/(g·K), whence $\Delta T \simeq 80$ K).

In the range $\dot\varepsilon > 7\cdot10^4$ sec^{-1} there is a sharp deterioration in metal ductility properties (rapid increase in Y) which leads to occurrence of a ductility peak on the $\varepsilon_p(\dot\varepsilon)$ curve (see Fig. 4).

We consider the second stage of failure, i.e., separation of the material into parts. By equating the elastic tensile energy to the energy going into forming free surface λ we obtain an equation for determining the number of fragments

$$Y^2 l/E = \lambda. \tag{13}$$

Substituting Eqs. (11) and (12) in (13) and using the equation $n = 2\pi b_p/\ell$, we find an equation for the number of fragments

$$n = \frac{2\pi b_0 Y_0^2}{E\lambda}\left(1 + \varepsilon_0 + \left(1 + \varepsilon_0 + \sqrt{3}\right)\frac{\eta v}{b_0 Y_0}\right)\left(1 + \frac{\eta v}{b_0 Y_0}\right). \tag{14}$$

Equation (14) is studied with velocities v < 1.6 km/sec since with high velocities the number of fragments ℓ is determined by shell thickness [3]. A typical feature of Eq. (14) is the clearly expressed scale effect $n(b_0)$ noted previously in [3]. Given in Fig. 6 is the dependence n(v) for a fixed shell radius b_0. The curve is described by Eq. (14) with $\eta = 3.4\cdot10^3$ Pa·sec, Y_0 = 0.4 GPa, E = 200 GPa, b_0 = 2.1 cm, ε_0 = 0.3, λ = 1 J/cm^2. Points 1 relate to an x-ray test, and 2 relate to tests for arresting fragments in sawdust [3]. It can be seen that theoretical curve (14) describes quite well tests for arresting fragments in sawdust. Some difference in x-ray tests is apparently connected with recording to fine fractions of fragments in these tests.

By using Fig. 6 we estimate the characteristic time for the second stage $t_2 \approx \ell/c$. Assuming that v = 1 mm/sec, from Fig. 4 we find ε_p = 0.78, and from Fig. 6 the number of fragments n ≃ 40. The size of a fragment is determined by the equation $\ell = 2\pi(1 + \varepsilon_p)b_0/n$, whence ℓ ≃ 0.58 cm, $t_2 \simeq 1.2$ µsec with c = 5 km/sec (c is speed of sound in steel). The time of shell expansion to failure for the case given $t^* = \varepsilon_p/\dot\varepsilon \simeq 16$ µsec. The value of t^* is an order of magnitude greater than t_2 which proves the correctness of the approximations made in deriving Eq. (4).

It should be noted that power dependence $Y(\dot\varepsilon)$ given in Fig. 5 is typical for "soft" metals. The sharp increase in Y with $\dot\varepsilon \sim 10^5$ sec^{-1} is connected first with limitation of dislocation velocity which does not exceed the speed of sound. For "hard" metals a linear dependence $Y(\dot\varepsilon)$ is typical with a high viscosity coefficient η [14]. Intense strengthening arises as a result of retardation of dislocations at impurities, grain boundary walls, and other lattice defects. The model suggested in this work for the ductility peaks relates to "soft" metals, and the model in [1] relates to "hard" metals. For "soft" metals it is necessary to produce higher plastic strain prior to accumulation of considerable damage since the ductility peak is connected with the first stage. In changing over to "hard" metals damage accumulate rapidly (e.g., as a result of the Stroh mechanism [15] and others) and the ductility peak is connected with the second stage, i.e., separation of the material into parts. This case case is described in the model in [1] which leads to higher values of η and λ than for the model given.

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ELASTOPLASTIC TORSION THEORY FOR WHISKER CRYSTALS

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1. Introduction. Plastic deformation of crystals is accompanied by the complex evolution of internal elastic stress fields caused by self-consistent movement of conglomerates of crystal defects at different structural levels of deformation [1]. A specific role in formation of these elastic fields applies to the crystal surface [2] which thus appears to be incorporated in a number of factors which affect movement of the defect structure. In view of this with the microscopic nature of some of the linear dimensions of a crystal in the kinetics of plastic strain the structural level of deformation, whose size is comparable with the size of a crystal, becomes decisive. In this respect whisker crystals (WC) are unique model objects for studying in a "pure form" features of the development of plastic deformation in assemblies of defects of different hierarchical degrees of structural levels. In an experimental respect torsion and bending are convenient methods for studying the ductility of WC [3], and the dislocation structure which forms is satisfactorily revealed by direct methods [4]. It is of interest to obtain general relationships which connect the macroscopic reaction of a WC (i.e., the amount of torsion) to dislocations present within it with characteristics of the dislocation structure which emerges as a basis for theoretical analysis of the plastic behavior of WC in torsion and also in the general case in bending.

In the present work in an approximation of macroscopically average description of the dislocation structure elastic torsion, and as a generalization bending, of whisker crystals caused by presence of dislocations in a crystal are considered. Relationships are found from the condition for minimum elastic energy which determine the macroscopic reaction of a whisker crystal to dislocations introduced into it.

2. Statement of the Problem. Finding actual elastic dislocation fields located close to a surface is a very complex mathematical problem [2] for which there is not yet a satisfactory solution. In particular, the problem of torsion for a WC containing dislocations is currently solved in a general form only for the case of rectilinear screw dislocations parallel to the crystal axis. As shown by Eshelby [2], the twist angle for WC ignoring edge effects at is ends is expressed in terms of the value of the Prandtl torsion function at

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